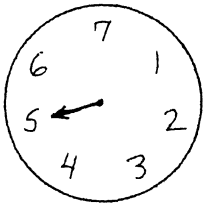
April 1, 1992 WOCOMAL VARSITY MEET

ROUND I: Elementary number theory

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. 

In the figure at the left, the hand pointing to 5 moves clockwise to the next numeral every hour. At the end of 24 hours, to which numeral will it point?

2. If the Least Common Multiple of 420 and 270 is divided by the Greatest Common Factor of 420 and 270, the result is what number?

3. When 7^9 is divided by 13, the remainder is 8. What is the remainder when 7^{10} is divided by 13?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Shepherd Hill, Tantasqua, Worcester Academy

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ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. When five consecutive integers are added, their sum is 155. Find the largest one.

2. If the ratio of a to b is R and $a \neq b$, express the ratio of $a+b$ to $a-b$ in terms of R.

3. If x is divided by y, the quotient is 8 and the remainder is 3. If 8 is divided by z, the quotient is 4 and the remainder is 1. If x is divided by yz, the remainder is $y+3$. What is the quotient?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(2 pts) 3. _____

Algonquin, Burncoat, Doherty

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ROUND III: Theory of polynomial equations and functions

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. If $f(x) = x^2 + 1$ and $g(x) = x - 1$, express $f(g(x))$ as a polynomial.

2. A cubic polynomial equation, $p(x) = 0$, has real coefficients and leading coefficient equal to 1. One of its roots is $1 + 2i$ and the sum of the roots is 14. Write $p(x)$ in polynomial form.

3. For how many integers $c \leq 50$ will the roots of the equation $x^2 - 4x + c = 0$ be complex conjugates of the form $a \pm bi$ where a and b are positive integers?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. $p(x) =$ _____

(3 pts) 3. _____

Quaboag, Tahanto, Tantasqua

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ROUND IV: Similarity and Pythagorean relationships

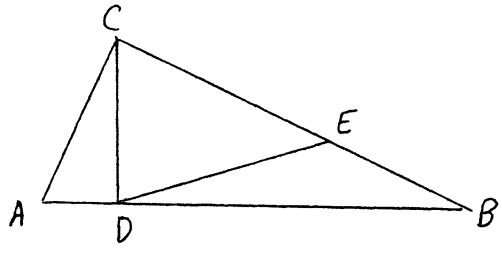
ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1 Triangle ONE has right angle N. If $ON = 9$ and $OE = 41$, find NE.

2. A person 5 ft 9 in tall casts a $3\frac{1}{2}$ ft shadow, the head end of which coincides with the end of the shadow cast by a pole 12 ft away from the person. How tall is the pole to the nearest half foot?

3 Triangle ABC is a right triangle with hypotenuse $\overline{AB}$, $\overline{CD} \perp \overline{AB}$, $AD = 4$, $DB = 12$, $BE = \frac{BC}{4}$.

Find the area of triangle DEB.



ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Leicester, Millbury, Notre Dame

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ROUND V: Trigonometry - open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. Find, in degrees, the smallest positive angle x for which

$$\left(2^{\sin^2 x}\right)\left(2^{\cos^2 x}\right)\left(2^{\tan^2 x}\right) = 2^2.$$

2. In right triangle ABC find the numerical value of

$$\sin^2 A + \sin^2 B + \sin^2 C.$$

3. In $\triangle PQR$, $\tan P = 0.75$ and $\tan Q = 2.4$. Find the value of $\frac{p}{q}$, where p and q are the lengths of the sides of the triangle opposite angles P and Q, respectively.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Shrewsbury, South, Tshanto

April 1, 1992 WOCOMAL VARSITY MEET

TEAM ROUND: Topics of previous rounds and open 2 points each

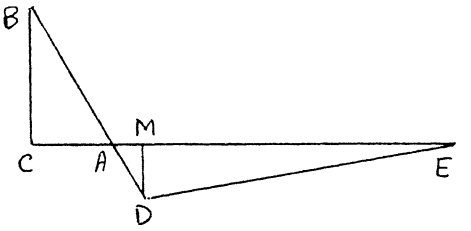
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ANSWER SHEET

1. Find the repeating decimal form of the base 10 decimal 0.25 in base 3. (Write your answer with a bar over the repeating part, as in $0.\overline{6}$.)

2. The average age of a group of doctors and lawyers is 40 years. If the average of the doctor's ages is 37 and that of the lawyer's ages is 50, find the ratio of the number of doctors to the number of lawyers.

3. Let $f(x) = ax^2 + bx + c$ with $f(0) = -\frac{1}{2}$, $f(2) = \frac{1}{2}$, and $f(4) = 3\frac{1}{2}$. Find $f(8)$.

4. Given: $\overline{BC} \perp \overline{AC}$, $\overline{DM} \perp \overline{CE}$, $AC = 8$, $BC = 15$, $AD = 5$, and area of $\triangle ABC$ = area of $\triangle ADE$. Find AE.

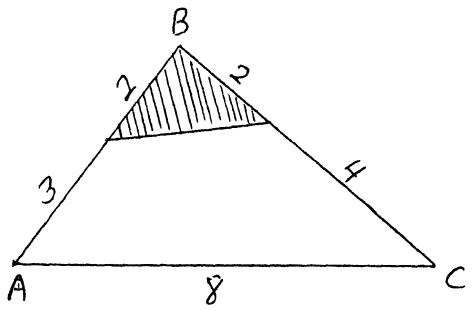


5. If $0 \le x \le \frac{\pi}{4}$ and $\sin(2x) = 0.6$, find the value of $\tan x$.

6. If $\frac{m}{n} = \frac{4}{3}$ and $\frac{r}{t} = \frac{9}{14}$, evaluate $\frac{3mr - nt}{4nt - 7mr}$.

7. If $z = x + y$, find the largest value of z for x and y satisfying all of these inequalities simultaneously: $y \ge 1$, $x \le 6$, and $y \le 2x + 1$.

8. For all $x > 0$ and all $y > 0$, the function f satisfies $f(xy) = f(x) + f(y)$. If $f(2) = a$ and $f(3) = b$, find $f(72)$ in terms of a and b.

9. 

What fraction of the area of $\triangle ABC$ is shaded?

Algonquin, Bromfield, Burncoat, Doherty, Marlboro, St. John's, Worcester Academy

April 1, 1992

ROUND I

I
1. 1
2. 126
3. 4

II
1. 33
2. $\frac{R+1}{R-1}$
3. 4

III
1. $x^2 - 2x + 2$
2. $x^3 - 14x^2 + 29x - 60$
3. 6

IV
1. 40
2. $25\frac{1}{2}$ ft or 25 ft 6 in
3. $6\sqrt{3}$

V
1. $45°$
2. 2
3. $\frac{13}{20}$ or 0.65

TEAM ROUND

1. $.\overline{02}_3$
2. 10:3 or $\frac{10}{3}$
3. $15\frac{1}{2}$ or...
4. $\frac{136}{5}$ or $27\frac{1}{5}$ or...
5. $\frac{1}{3}$
6. $-\frac{11}{14}$
7. 19
8. $3a + 2b$
9. $\frac{2}{15}$